Analysis of Eddy Currents in Litz Wire Using Homogenization-based FEM

Shingo Hiruma[1], Hajime Igarashi[1], *Member, IEEE*

[1]Graduate School of Information Science and Technology, Hokkaido University, 060-0814 Sapporo, Japan

In this work, Litz wire composed of stranded wires is modeled as anisotropic material with the macroscopic complex permeability to analyze skin and proximity effects by the finite element method (FEM). In the proposed method, the permeability tensor of the strand is computed at each element considering the wire direction. The computational burden can be greatly reduced by the proposed method because it does not require fine spatial discretization unlike conventional FEM.

***Index Terms*—Complex permeability, homogenization method, eddy current, Ollendorff formula, Litz wire**

I. Introduction

As the driving frequency of electric apparatus increases, the eddy current losses due to skin and proximity effects become more significant. To reduce these losses, Litz wires are widely used in transformers, wireless power transfer devices and so on. Because the Litz wire is composed of a number of strands, it would take unacceptably long time to analyze the eddy currents when using conventional FEM.

Using homogenization-based FEM [1-3], the eddy current losses in multi-turn coils can effectively be evaluated because the coil region is modeled as a uniform material with the macroscopic complex permeability for which we do not need fine discretization. However, in this method, the coils are assumed to be in parallel. We have not been able to apply this approach to the analysis of the Litz wires which are composed of fine strands. The authors have proposed an approach based on the one-dimensional integral equation [4]. In this method, the wires are modeled as polygonal lines so that the skin and proximity effects in the Litz wire can be analyzed. However, the resulting linear equation includes a dense matrix. Thus we need special techniques such as the fast multipole expansion and H-matrix method to use this method when the Litz wires are composed of more than some hundred strands.

In this work, we extend the homogenization-based FEM to analyze the eddy currents in the Litz wire by introducing the tensorial macroscopic complex permeability. It will be shown that the proximity loss of the Litz wire calculated by this method is in good agreement with that computed by the integral equation method.

II. Formulation

A. Complex permeability

Let us consider an isolated round wire, radius a, conductivity σ, relative permeability μ_0, immersed in a time-harmonic magnetic field of angular frequency ω. The curvature of the wire is assumed negligible. Then the eddy currents due to the proximity effect are obtained by analytically solving the Helmholtz equation. The dipole magnetic field induced by the proximity effect is anti-parallel to the external field outside the wire. This diamagnetic property can be represented by the complex permeability [2]

$$\dot{\mu} = \mu_0 \frac{J_1(z)}{z J_1'(z)} \tag{1}$$

where $z = a\sqrt{-j\omega\sigma\mu_0}$ and J_1 is the first-order Bessel function. Note that $\dot{\mu}$ is defined in the plane perpendicular to the wire direction. Because there is almost no diamagnetic field along the wire direction, the permeability along the wire can be set to μ_0.

B. Macroscopic permeability

Now we consider the multi-turn coils in which the wires arranged in parallel. By using the Ollendorff formula [2], we can obtain the macroscopic complex permeability given by

$$\langle \dot{\mu} \rangle = \mu_0 + \frac{2\eta(\dot{\mu}-1)}{2+(1-\eta)(\dot{\mu}_r-1)} \tag{2}$$

where η is filling rate of the wires. We can model the coil region composed of the wires and air with the homogenized uniform material with $\langle \dot{\mu} \rangle$.

Then we consider the Litz wire which is composed of multiple strands. In this case, the complex permeability would depend on the position in the coil region due to the wire transposition. To model this structure, we assume that the wires are locally arranged in parallel, and the macroscopic complex permeability can be defined on the plane perpendicular to the wires. In the coordinate system L (u, v, w) shown in Fig.1, the permeability tensor is given by

$$\mu_L = \begin{bmatrix} \langle \dot{\mu} \rangle & 0 & 0 \\ 0 & \langle \dot{\mu} \rangle & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \tag{3}$$

In the FE analysis, the permeability should be represented in the reference coordinate system G (x, y, z) shown in Fig.1. Therefore, the permeability μ_L should be transformed to

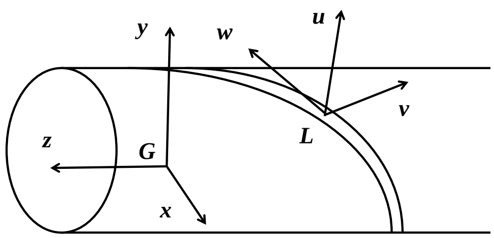

Fig. 1. Coordinate system of the twisted wires

permeability μ_G. Transformation matrix K: $L(u, v, w) \rightarrow G(x, y, z)$ [5] is defined as

$$K = R_z(\theta_1) R_x(\theta_2) \tag{4}$$

where R_x, R_z represent the rotational matrix around the x and z axes, respectively. In (4), the angles θ_1, θ_2 are defined as

$$\theta_1 = \tan^{-1} \frac{y}{x} \tag{5}$$

$$\theta_2 = \tan^{-1} \frac{2\pi\sqrt{x^2 + y^2}}{L_s} \tag{6}$$

where L_s is the twist pitch. Using this transformation matrix, the tensor permeability μ_G is represented as

$$\mu_G = K \mu_L K^{-1} \tag{7}$$

When analyzing the Litz wire by FEM, the permeability tensor (7) is evaluated at the center of each element.

III. Computation Results

We apply the proposed method to the analysis of the homogenized model of parallel and multiple strands shown in Fig.2. The original model consists of 49 wires whose radius is 0.15 mm and conductivity is 5.76×10^7 S/m, $\mu = \mu_0$ and the wire pitch is 15 mm. When using proposed method, we model it as one homogenized conductor. The filling rate η is 0.49, and the permeability of the conductor is calculated from (7). The current is assumed to be in parallel to the wire axis which corresponds to the z-axis in Fig.1, for simplicity.

We impose the periodic boundary condition on the faces ABCD and EFGH because the magnetic field is not parallel to these faces when the wires are twisted.

Assuming that the total current is 49 A, the proximity losses of parallel and multiple strands are calculated using the proposed method and one-dimensional integral equation method [4]. The imaginary part of the magnetic flux density, which represents the magnetization due to the eddy currents, is shown in Fig.3. We can see in Fig.3 that the proposed method enables to represent the twisted structure. The frequency characteristic of the proximity losses is plotted in Fig.4. We can see in Fig.4 that the result of proposed method is in good agreement with that obtained by the integral equation method. The error between them would be due to the fact that the actual external current is no parallel to the wire axis.

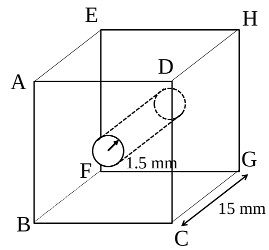

Fig. 2. Analysis model

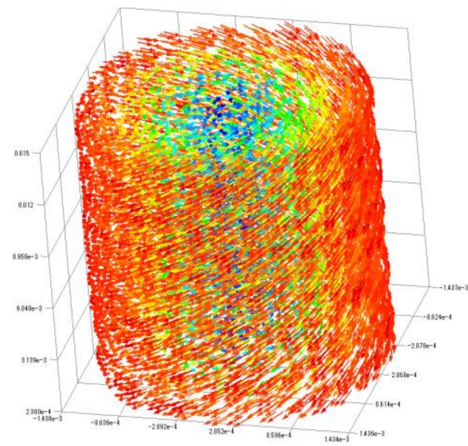

Fig.3 Magnetic flux density in multiple strands

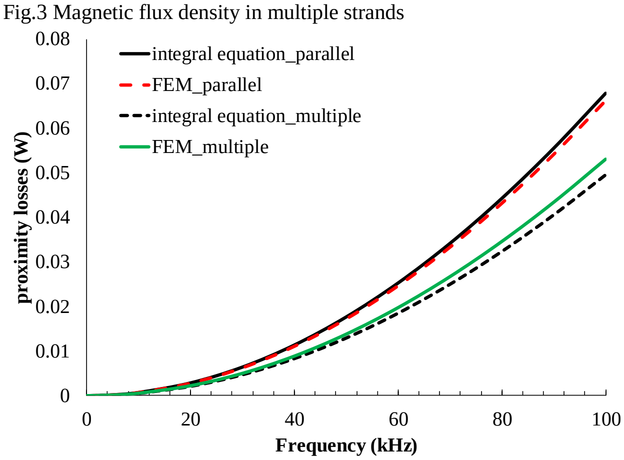

Fig.4. Frequency characteristic of proximity effect loss

References

[1] A. D. Podoltsev, I. N. Kucheryavaya, and B. B. Lebedev, "Analysis of effective resistance and eddy-current losses in multiturn winding of highfrequency magnetic components," *IEEE Trans. Magn*., vol. 39, no. 1, pp. 539-548, Jan. 2003.

[2] H. Igarashi, "Semi-Analytical Approach for Finite Element Analysis of Multi-turn Coil Considering Skin and Proximity Effects," *IEEE Trans. Magn*., vol53, no.1, 7400107, 2017

[3] Y. Sato, H. Igarashi, "Homogenization method based on model order reduction for FE analysis of multi-turn coil," presented at CEFC2016, submitted to *IEEE Trans. Magn*., 2016.

[4] S. Hiruma, H. Igarashi, "Fast three-dimensional analysis of eddy current in Litz wire using integral equation," presented at CEFC2016, submitted to *IEEE Trans. Magn*., 2016.

[5] N. Takahashi, T. Nakata, Y. Fuji, K. Muramatsu, M. Kitagawa, J. Take-hara, "3-D finite element analysis of coupling current in multifilamen-tary AC superconducting cable," *IEEE Trans. Magn*., vol. 27, No.5, pp. 4061-4064,1991.